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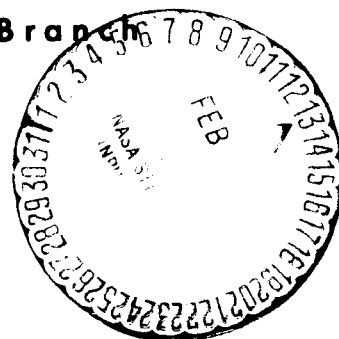
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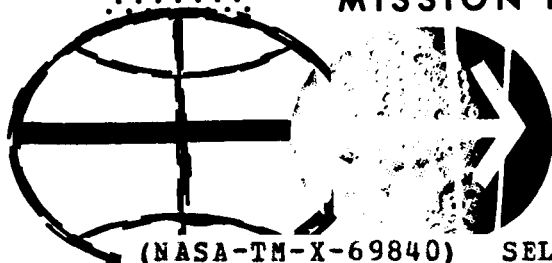
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SELECTION AND VERIFICATION  
OF USBS DOPPLER DATA SOURCES  
DURING LM ASCENT AND DESCENT

By G. Edward Wilson  
and  
Alan D. Wylie  
Mathematical Physics Branch



MISSION PLANNING AND ANALYSIS DIVISION



MANNED SPACECRAFT CENTER  
HOUSTON, TEXAS

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DURING LM ASCENT AND DESCENT (NASA)

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PROJECT APOLLO

SELECTION AND VERIFICATION OF USBS DOPPLER DATA  
SOURCES DURING LM ASCENT AND DESCENT

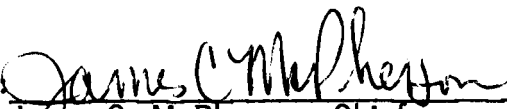
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MANNED SPACECRAFT CENTER  
HOUSTON, TEXAS

Approved: \_\_\_\_\_

  
James C. McPherson, Chief  
Mathematical Physics Branch

Approved: \_\_\_\_\_

John P. Mayer, Chief  
Mission Planning and Analysis Division

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## SELECTION AND VERIFICATION OF USBS DOPPLER DATA SOURCES

### DURING LM ASCENT AND DESCENT

By G. Edward Wilson and Alan D. Wylie

### SUMMARY AND INTRODUCTION

This note describes a program for verifying and selecting USBS Doppler data sources during LM ascent and descent. It also provides equations for selecting the primary guidance and navigation control system (PGNCS) and abort guidance system (AGS) telemetered vector sources. The assumption is made that three USBS ground stations are simultaneously tracking the LM during ascent and descent. One station will transmit and receive two-way, nondestruct Doppler data. One or two other stations will receive three-way Doppler data. A further assumption is made that the Doppler biases have been solved for.

In order to check the validity of the data, residuals for data from one station will be compared with residuals for the other two stations. The most probable case is that all three residuals will agree to within some expected tolerance. However, if two stations agree and a third disagrees, the data from the station in disagreement will be discarded.

When at least two Doppler sources are verified, the program will proceed to the telemetry vector source selection. Residuals will be computed using the telemetry vector not previously used in data verification. The residuals for the stations using one vector source will be averaged and compared with the average of the residuals using the other vector source.

### GENERAL PROCEDURE FOR SELECTING AND VERIFYING DOPPLER DATA SOURCES

For clarity, figure 1 shows only two-way Doppler for station 1. The discussion, however, will be generalized to include three stations.

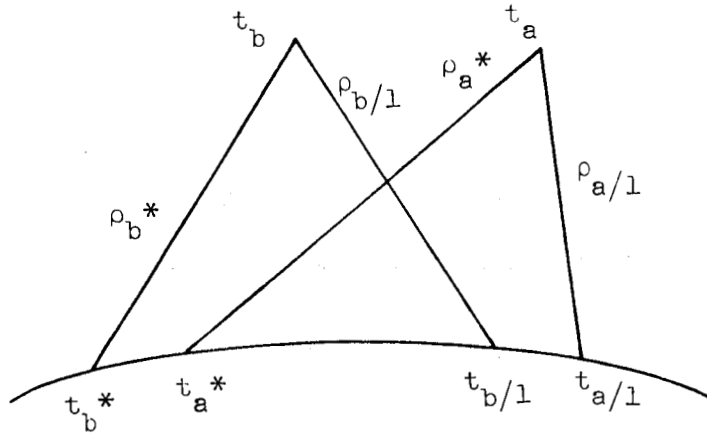


Figure 1.- Station-vehicle geometry.

Let the  $N_{b/i}$  and  $N_{a/i}$  be the nondestruct N count observables at times  $t_{b/i}$  and  $t_{a/i}$ , respectively, at station  $i$  ( $i = 1, 3$ ). Station 1 is assumed to be the transmitter (see fig. 1), transmitting the two signals at  $t_b^*$  and  $t_a^*$ , respectively. Let  $t_b$  and  $t_a$  be the times the signals are transmitted from the vehicle. Let the  $\rho_{b/i}$  and  $\rho_{a/i}$  be the downlink ranges, and  $\rho_b^*$  and  $\rho_a^*$  the uplink ranges, anchored at times  $t_b$  and  $t_a$ , respectively, and used for the computed Doppler frequency,  $D_i$  (see p. 10). Let  $S_i$  be the pseudoactual

Doppler frequency shift,  $\frac{N_{a/i} - N_{b/i}}{t_{a/i} - t_{b/i}}$ . The residuals for each of the three stations can then be computed by

$$\Delta Y_1 = S_1 - D_1,$$

$$\Delta Y_2 = S_2 - D_2,$$

and

$$\Delta Y_3 = S_3 - D_3.$$

These  $\Delta Y_i$  will be displayed on a plot as depicted in figure 2.

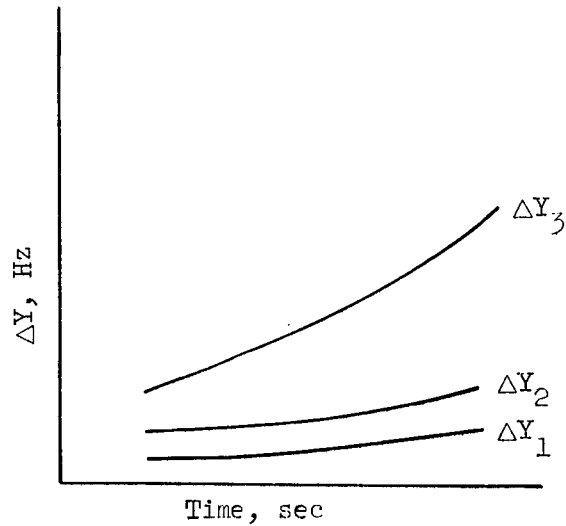


Figure 2.- Station verification display.

For the vector source selection, the residuals  $\Delta Y_i$  are then computed using the telemetry vector source not previously used. Each set of residuals is then averaged as follows:

$$\delta Y = \frac{\sum_{\ell=1}^{n'} \Delta Y_{\ell}}{n'}$$

where  $n'$  is the number of stations verified (that is, either  $n' = 2$  or  $n' = 3$ ).

The  $\delta Y$  for PGNCs and AGS will then be displayed as in figure 3.

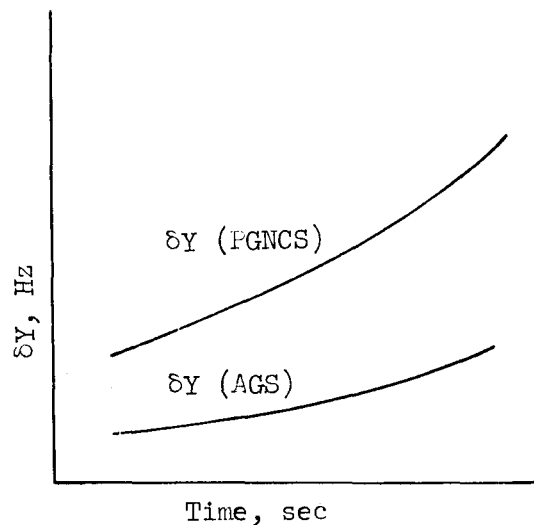


Figure 3.- Vector evaluation display.

#### DETAILED EQUATIONS AND LOGIC

##### Main Program

The program for selecting and verifying Doppler data sources is broken down into four parts: the main program or driver, the iteration subroutine, the interpolation subroutine, and the residual computation subroutine. Flowcharts 1 through 4 present the equations and logic for each of the four parts of the program.

All variables must be initialized to zero at the start of the program. The orbit determination constants described in the appendix are then read in. In addition, station characteristics and a lunar ephemeris must be available. Tables of time versus edited Doppler, N-count observations are generated for  $n$  stations. Let  $R(t_a)$  be the selenocentric position vector of the vehicle at time  $t_a$ . The program tests whether this vector is PGNCs or AGS and then checks if its time tag has been updated since the time of the preceding vector from the same source. If the last test fails, the program recycles to pick up another vector.

If the time tag has been updated, the iteration subroutine is called, the outputs of which are  $\rho_{a/i}$ ,  $\rho_a^*$ , and  $t_{a/i} - n$  downlink ranges, an uplink range, and  $n$  times which represent the times the observations would arrive at the receiving stations if the signal had left the vehicle at  $t_a$ . The last  $M$  observations will then be copied from the observation tables being generated for the  $n$  stations tracking the vehicle into the following arrays:  $t_{i/j}$  and  $N_{i/j}$  where  $i = 1, n$  and  $j = 1, M$ . The variable  $M$  will be determined by the sample rate of the Doppler observations, such that  $M$  observations encompass 10 seconds; for example, if the sample rate is 5 observations/second, then  $M = 50$ . Two time tests are then performed, the first to determine the validity of the telemetry vector time tag and the second to determine if  $t_{a/i}$  obtained from the iteration routine lies within the range of the tables that were copied for station  $i$ .

When the  $t_{i/j}$  and  $N_{i/j}$  are properly set up, the interpolation subroutine is called yielding  $N_{a/i}$ , the interpolated value of the count for station  $i$  at each  $t_{a/i}$ . The program checks whether the telemetry vector  $R(t_a)$  is PGNCS or AGS and then passes the appropriate times, ranges, and interpolated counts to the residual computation subroutine.

Outputs of the residual computation subroutine are  $\Delta Y_1$ , the residuals computed using one vector source and the interpolated counts from each of the  $n$  stations. The  $\Delta Y_1$  are then displayed as shown in figure 2. The average residual  $\delta Y$  computed by using one vector source is then compared on a display with the average residual of the other vector source as shown in figure 3. The number of stations verified is  $n'$ . The current values of time tag, observation time, uplink and downlink ranges, and Doppler count are then stored, and the program recycles by reading in a new telemetry vector.

#### Iteration Subroutine (I)

Inputs to the iteration subroutine are the telemetry position vector,  $R(t_a)$ , in a selenocentric coordinate system with the X-Y plane in the mean equatorial plane, the X axis through Aries at the beginning of the nearest Besselian year (NBY), and the Z axis directed along the mean pole; a time tag,  $t_a$ , referenced to lift-off; station characteristics for station  $i$  ( $i = 1, n$ ); and a lunar ephemeris. The first computation



transforms the mean NBY vector to a true-of-date earth-centered inertial (ECI) vector by

$$R'(t_a) = (NP) \left[ R(t_a) + TR \right]$$

where TR is the translation from selenocentric coordinates to ECI coordinates and where (NP) is the precession-nutation matrix (see ref. 1). The subroutine first computes the uplink range by iterating to find the time,  $t_a^*$ , the signal would be transmitted from station number 1

to arrive at the vehicle at  $t_a$ . Characteristics from the station characteristics tape are obtained for station number 1, which is the transmitter. The iteration equation for the uplink time is

$$t' = t_a - \frac{\rho}{c}, \quad (1)$$

where  $c$  is the velocity of light and

$$\rho = \left| R'(t_a) - R_s(t') \right|, \quad (2)$$

where  $R'(t_a)$  is the geocentric inertial position vector of the vehicle at  $t_a$  in a true-of-date coordinate system, and  $R_s(t')$  is the geocentric inertial position vector of station number 1 at the computed transmission time,  $t'$ . For an initial guess of  $R_s(t')$ , we use  $R_s(t_a)$ ; thus,

$$\rho = \left| R'(t_a) - R_s(t_a) \right|. \quad (3)$$

We may then use  $\rho$  of equation (3) in equation (1) for an initial estimate of  $t'$ . The position of the station is then computed for the new  $\rho$  in equation (2). This  $\rho$  is then used in equation (1) to obtain a new  $t'$ . This iteration process is repeated until the difference between successive values of  $t'$  is less than or equal to  $10^{-9}$  seconds or until six iterations have been made.

A refraction correction is then applied to the computed uplink range in the following manner (see ref. 2). Let

$$R = \left| R_s(t') \right|$$

and

$$\bar{\rho} = R'(t_a) - R_s(t').$$

Then

$$\rho' = \rho + \frac{\bar{n}\bar{k}R\rho}{R_s(t') \cdot \bar{\rho}} \quad (4)$$

where  $\bar{n}$ ,  $\bar{k}$  are available from the station characteristic table (see appendix). The uplink range,  $\rho'$ , will then be stored as  $\rho_a^*$ .

The subroutine then computes downlink ranges and times for the  $n$  stations successively. The only change from the uplink computations is in the time-iteration equation which now becomes

$$t' = t_a + \frac{\rho}{c} \quad (5)$$

where  $t'$  is the computed time at which the signal should be received at the station if it had left the vehicle at  $t_a$ . After the refraction correction has been applied to a downlink range,  $\rho'$  is stored as  $\rho_{a/i}$  and  $t'$  stored as  $t_{a/i}$  ( $i = 1, n$ ) and the subroutine picks up the proper station characteristics for the downlink computation for the next station, restrictions being that station number 1 be the transmitter and that the ordering of stations 2 through  $n$  be maintained.

The position of the station,  $R_s(t')$ , is computed for each successive iteration of time,  $t'$ , from the station characteristics as follows:

Let  $\theta$  be the angle between the true equinox of date coordinate system and Greenwich at  $t'$ . Then

$$\theta = g_1 + g_2 T + g_3 T^2 + \delta\alpha + \omega \text{ (UTL)}$$

where  $UTL = t' - t_{LO} + DT$  and  $t_{LO}$  is the time from lift-off to

midnight of the day of lunar ascent; the other constants are given in the appendix.

$\lambda' = \lambda + \theta$  where  $\lambda$  is the longitude of the station.

Then,  $R_s(t')$  is given by

$$R_s(t') = \begin{bmatrix} \cos \lambda' (r \cos \phi' + h \cos \phi) \\ \sin \lambda' (r \cos \phi' + h \cos \phi) \\ r \sin \phi' + h \sin \phi \end{bmatrix}.$$

#### Interpolation Subroutine (II)

The interpolation subroutine is called to find a pseudoactual Doppler count,  $N_{a/i}$ , for each station at some time  $t_{a/i}$  which is not a recorded observation time. Inputs to the subroutine are  $t_{a/i}$  (obtained from the iteration routine) and  $n$  tables of  $M$  observation times,  $t_{i/j}$ , versus  $M$  observed Doppler counts,  $N_{i/j}$ , ( $i = 1, n$  and  $j = 1, M$ ). (It should be noted that the quantities in  $N_{i/j}$  are the raw, edited Doppler accumulated counts from count initialization and not a count difference over some fixed time interval.) The interpolation scheme tests the  $t_{a/i}$  of each station against the corresponding table of recorded observation times to find between which two recorded observation times  $t_{a/i}$  lies and, hence, between which two recorded observations  $N_{a/i}$  lies.

To find  $N_{a/i}$  the interpolation scheme proceeds as follows: Test  $t_{a/i}$  against successive values of  $t_{i/k}$  where  $k = M, M - 1, M - 2, \dots$ . Continue testing until

$$t_{a/i} > t_{i/k}.$$

Then test successive times,  $t_{i/k}$  and  $t_{i/k+1}$ , to see if

$$t_{i/k+1} - t_{i/k} > \sigma_T.$$

If so, the time interval is too long to linearly interpolate. Therefore, do not interpolate, but set  $N_{a/i}$  equal to a dummy negative value denoted by  $\sigma_N$ , and start the time tests for the next station. If, however,

$$t_{i/k+1} - t_{i/k} \leq \sigma_T,$$

the time interval is sufficiently short to accurately interpolate for  $N_{a/i}$ . Therefore, the following equation can be solved for  $N_{a/i}$ .

$$N_{a/i} = N_{i/k} + \frac{N_{i/k+1} - N_{i/k}}{t_{i/k+1} - t_{i/k}} (t_{a/i} - t_{i/k})$$

where  $i$  is the station number. The outputs,  $N_{a/i}$ , from the interpolation subroutine will then be passed to the main program.

### Residual Subroutine (III)

Inputs to the residual computation subroutine are two times, four ranges, two interpolated N-count observations, and a table of bias values. The subroutine will be called in two different places in the main program depending on whether the vector source is PGNCs or AGS; this selection is controlled by LSW (see appendix). The inputs to this subroutine can be controlled by the arguments of the calling statements; for example, for a PGNCs vector we might have:

CALL RESIDUAL (Pt<sub>b/i</sub>, t<sub>a/i</sub>, P<sub>p<sub>b</sub></sub>\*, P<sub>p<sub>b/i</sub></sub>,

p<sub>a</sub>\*, p<sub>a/i</sub>, PN<sub>b/i</sub>, N<sub>a/i</sub>, ΔY<sub>i</sub>, δY)

Then for an AGS vector we would have:

CALL RESIDUAL ( $At_{b/i}$ ,  $t_{a/i}$ ,  $A\rho_b^*$ ,  $A\rho_{b/i}$ ,

$\rho_a^*$ ,  $\rho_{a/i}$ ,  $AN_{b/i}$ ,  $N_{a/i}$ ,  $\Delta Y_i$ ,  $\delta Y$ )

Once the input arguments have been established, the program processes the residual subroutine, which is of the following form:

SUBROUTINE RESIDUAL ( $t_1$ ,  $t_2$ ,  $\rho_1$ ,

$\rho_2$ ,  $\rho_3$ ,  $\rho_4$ ,  $N_1$ ,  $N_2$ ,  $\Delta Y$ ,  $\delta Y$ )

where the variables other than  $\rho_1$ ,  $\rho_3$ , and  $\delta Y$  have three dimensions.

Before computing a residual, a test must be made using the computed observation times for the transmitting station to check if the time interval,  $t_2 - t_1$ , is less than TMAX, where TMAX is the maximum  $\tau$  interval for which the Doppler frequency is allowed to be computed. If this test fails, the routine returns to the main program to pick up the next telemetry vector. If  $t_2 - t_1$  is less than TMAX, then the validity of the two Doppler counts,  $N_1$  and  $N_2$ , at station  $i$  must be verified. A negative value for either  $N_1$  or  $N_2$  causes the residual computation to be omitted, and the routine checks the Doppler counts for the next station.

The computed Doppler observation is obtained from the following equation:

$$D_i = (\omega_3 + b_i) + \frac{\omega_4 f_{tr}}{c(t_{2/i} - t_{1/i})} \left[ (\rho_3 + \rho_{4/i}) - (\rho_1 + \rho_{2/i}) \right]$$

where the constants  $\omega_3$ ,  $\omega_4$ ,  $f_{tr}$ ,  $c$ ,  $b_i$  are explained in the appendix.

A pseudoactual Doppler observation,  $S_i$ , is obtained by

$$S_i = \frac{N_{2/i} - N_{1/i}}{t_{2/i} - t_{1/i}} ,$$

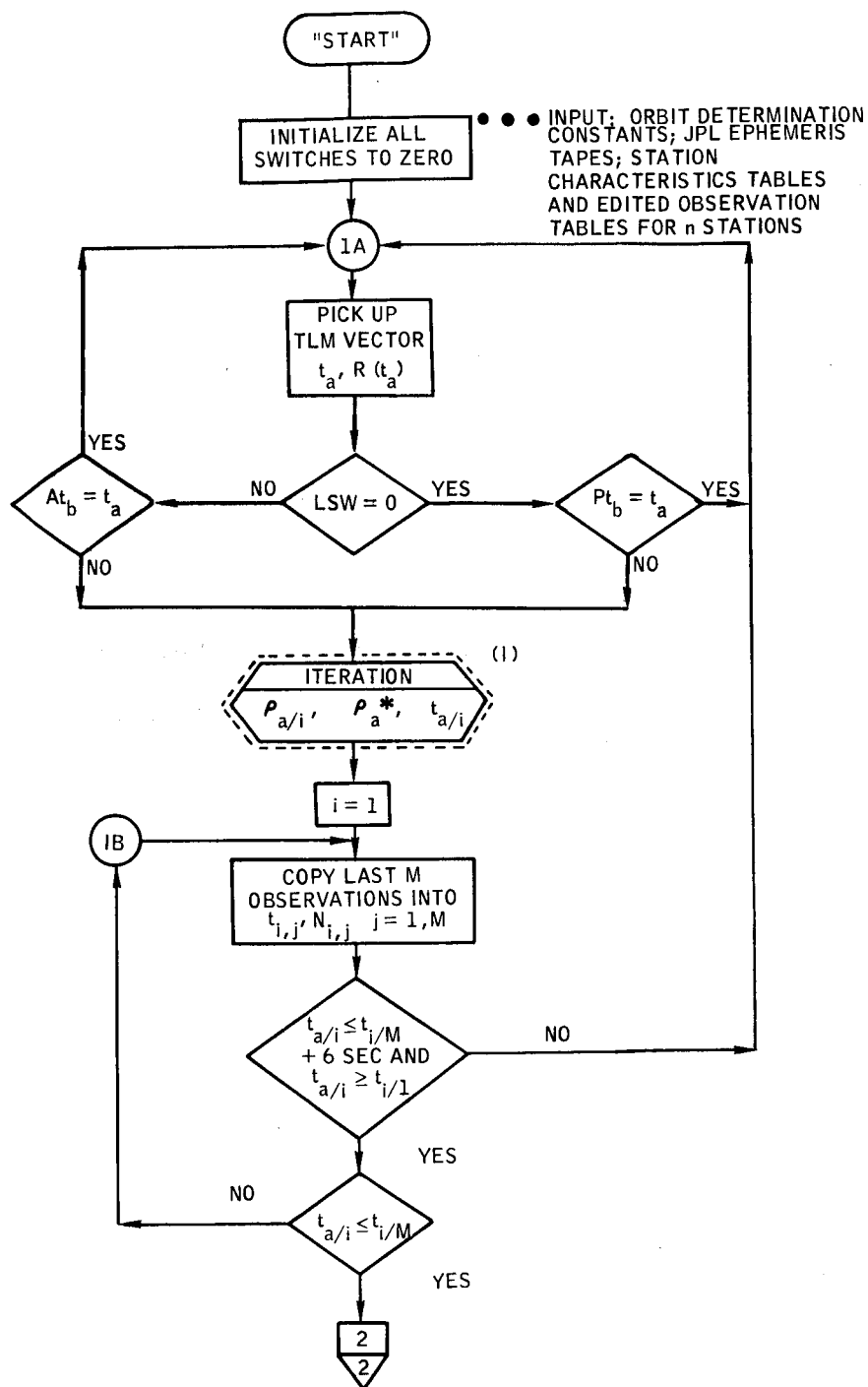
and the residual is computed from

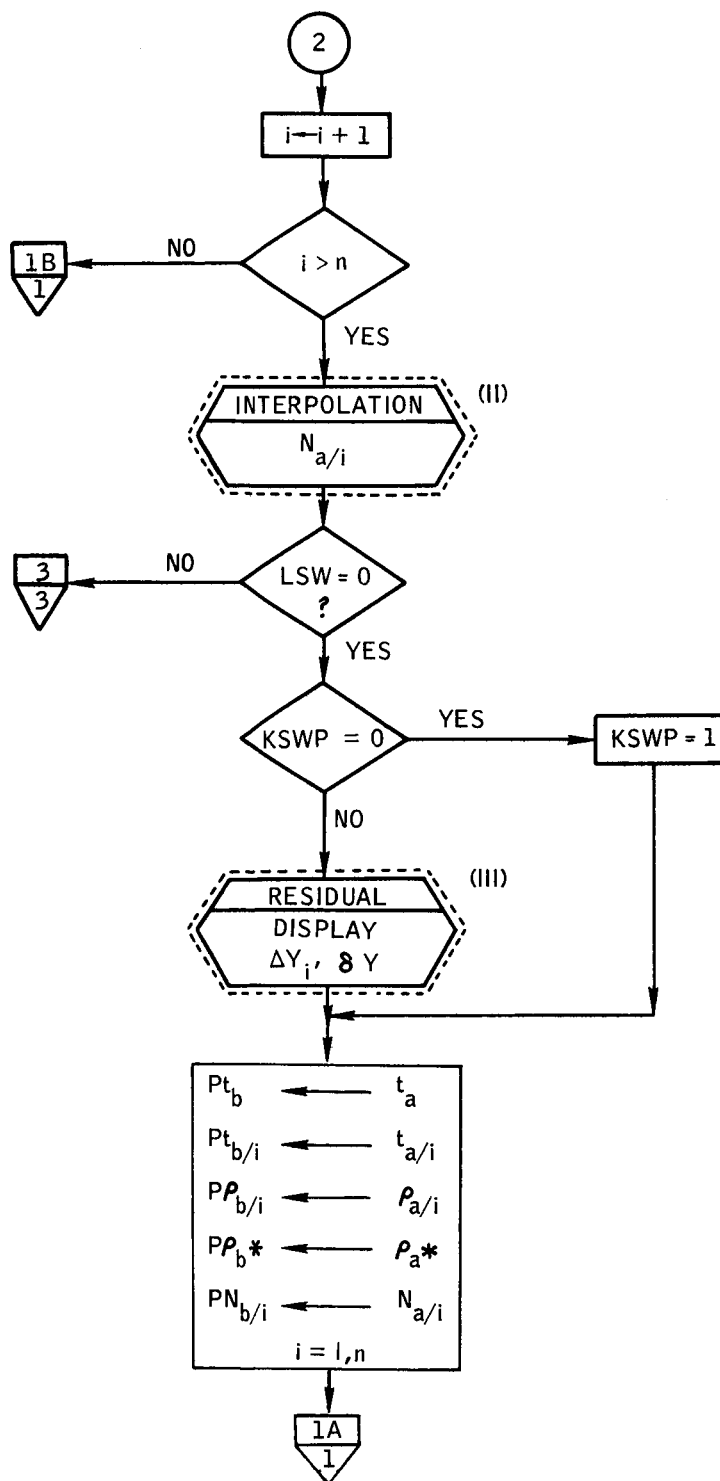
$$\Delta Y_i = S_i - D_i .$$

These  $\Delta Y_i$  are plotted as in figure 2. The average of the residuals from  $n'$  stations are then obtained by

$$\delta Y = \frac{\sum_{\ell=1}^{n'} \Delta Y_{\ell}}{n'}$$

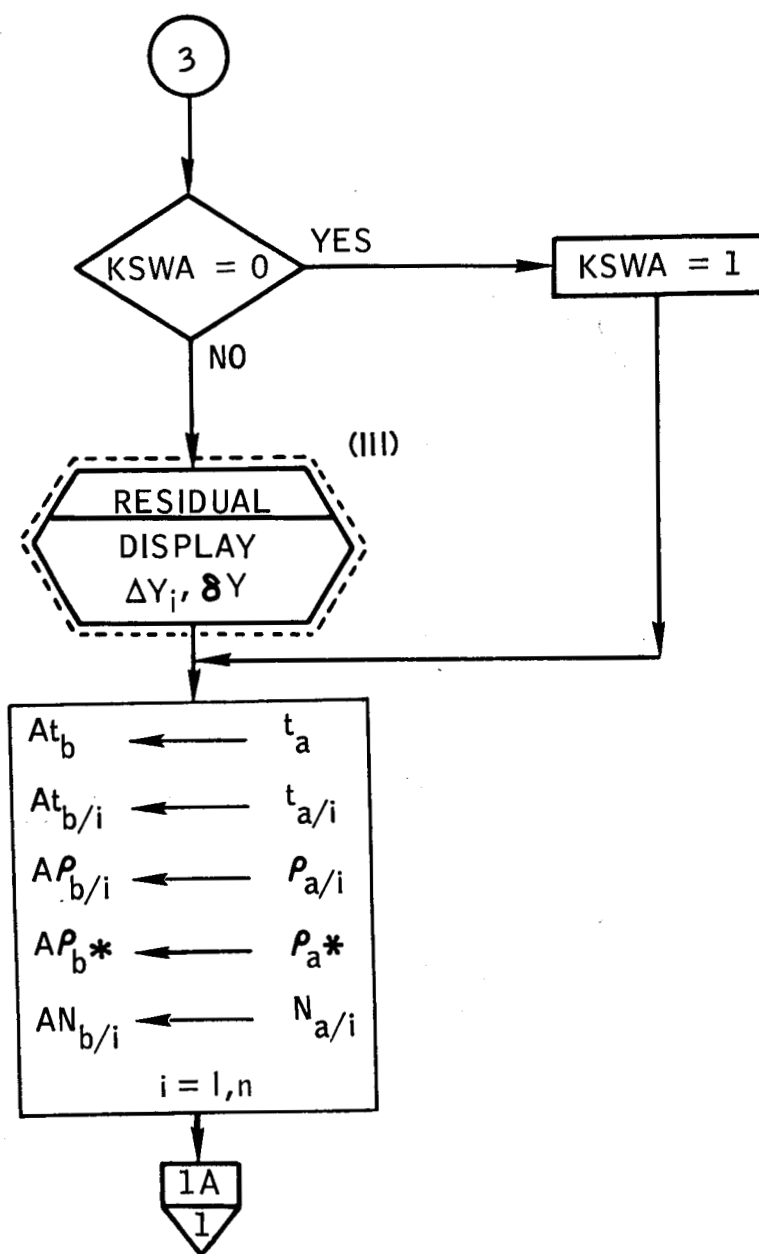
where  $n'$  is the number of stations verified. This  $\delta Y$  will then be plotted against  $\delta Y$  computed with the other vector source and displayed as shown in figure 3. Control is then transferred to the main program, and the current values of  $t_{a/i}$ ,  $\rho_a^*$ ,  $\rho_{a/i}$ ,  $N_{a/i}$  are saved for the next cycle for a particular vector source.



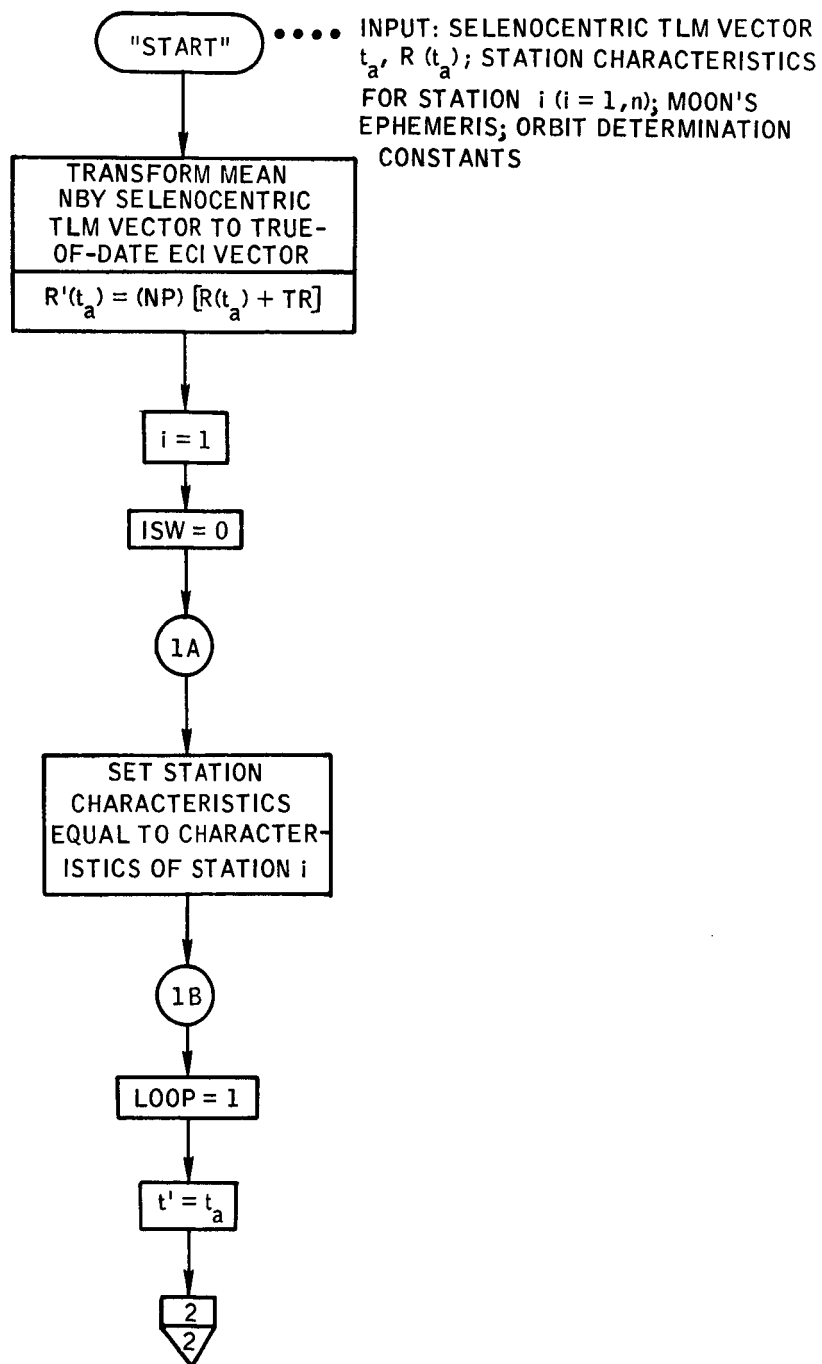


Flowchart 1.- Continued

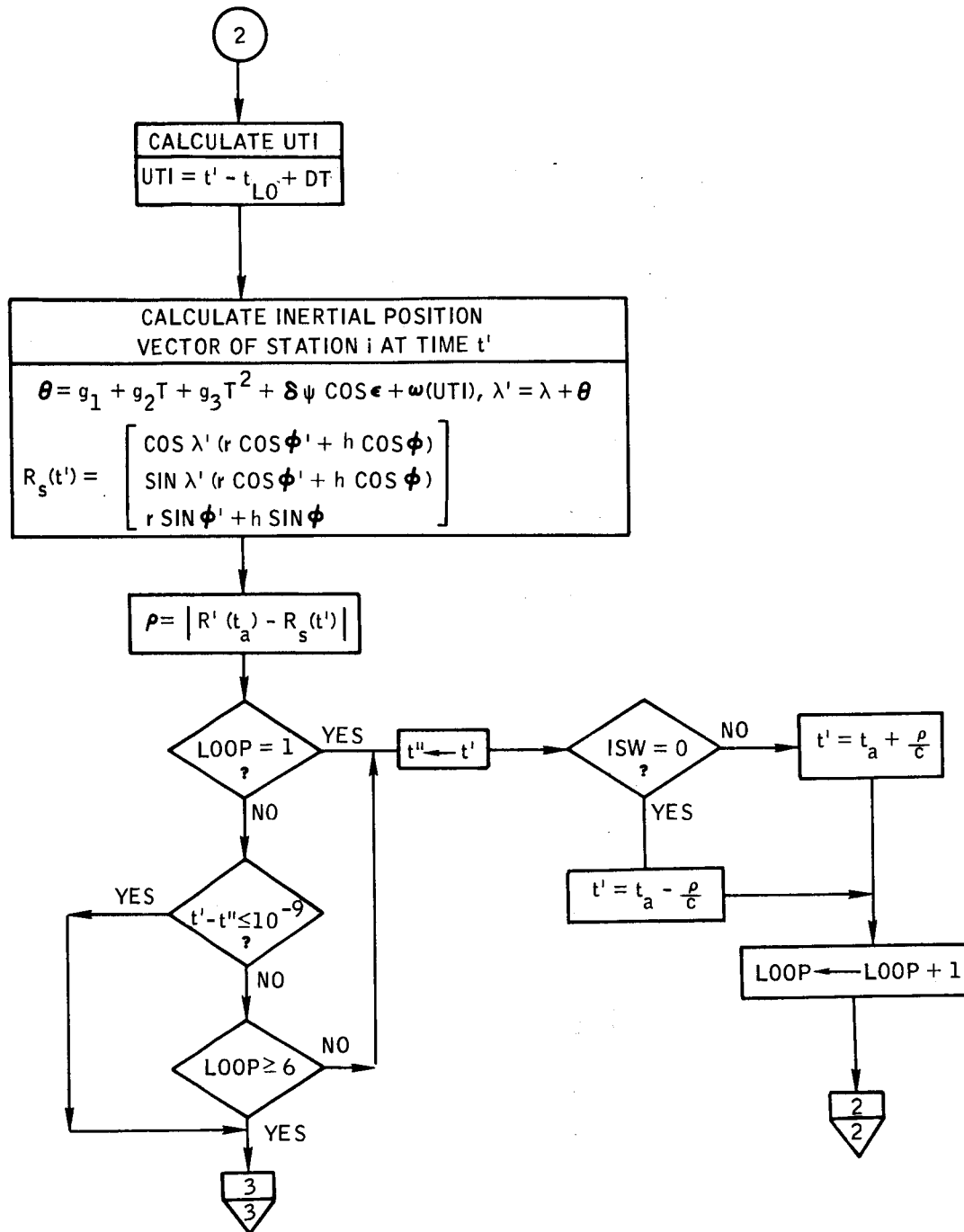




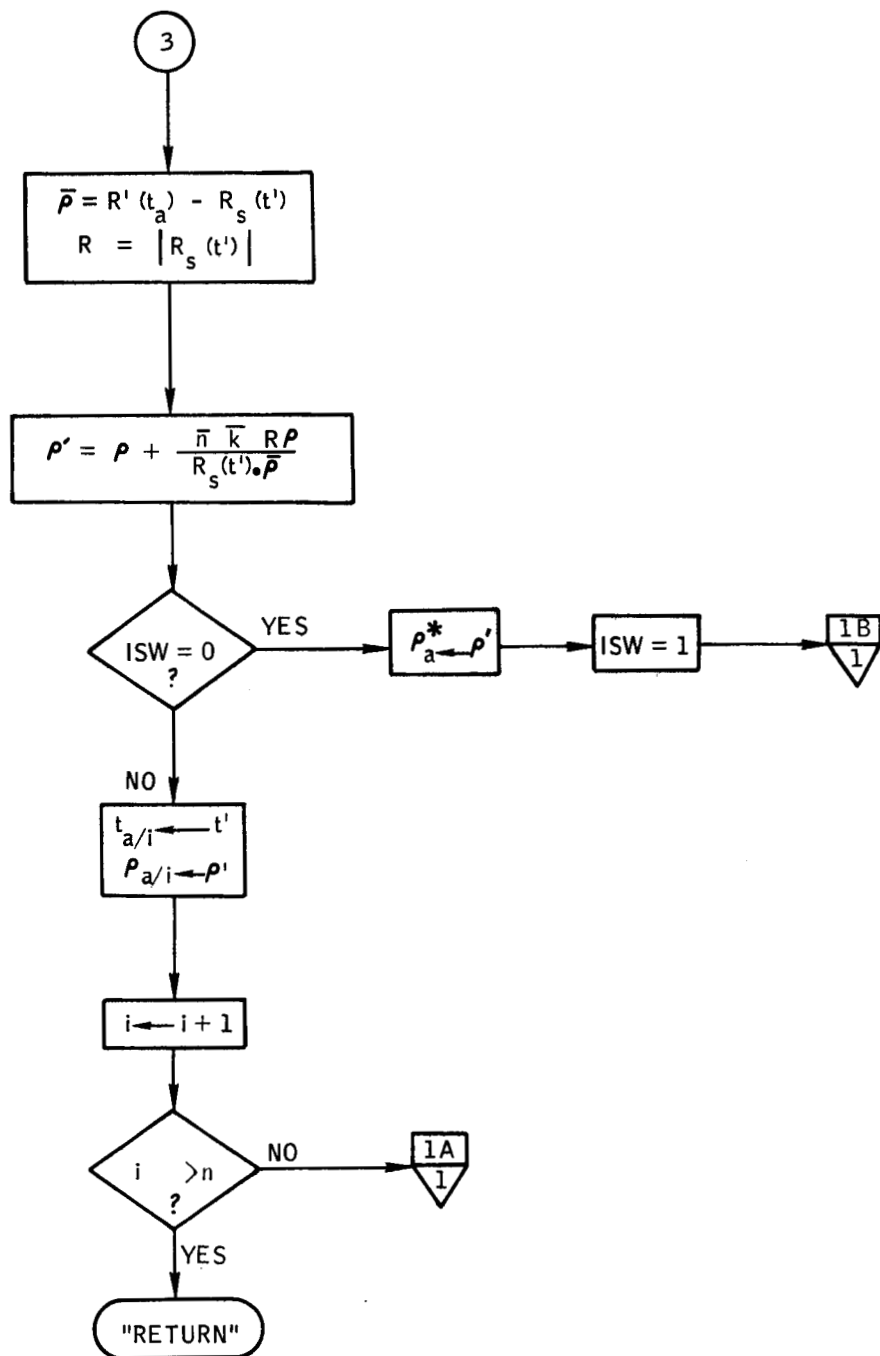
Flowchart 1.- Concluded.



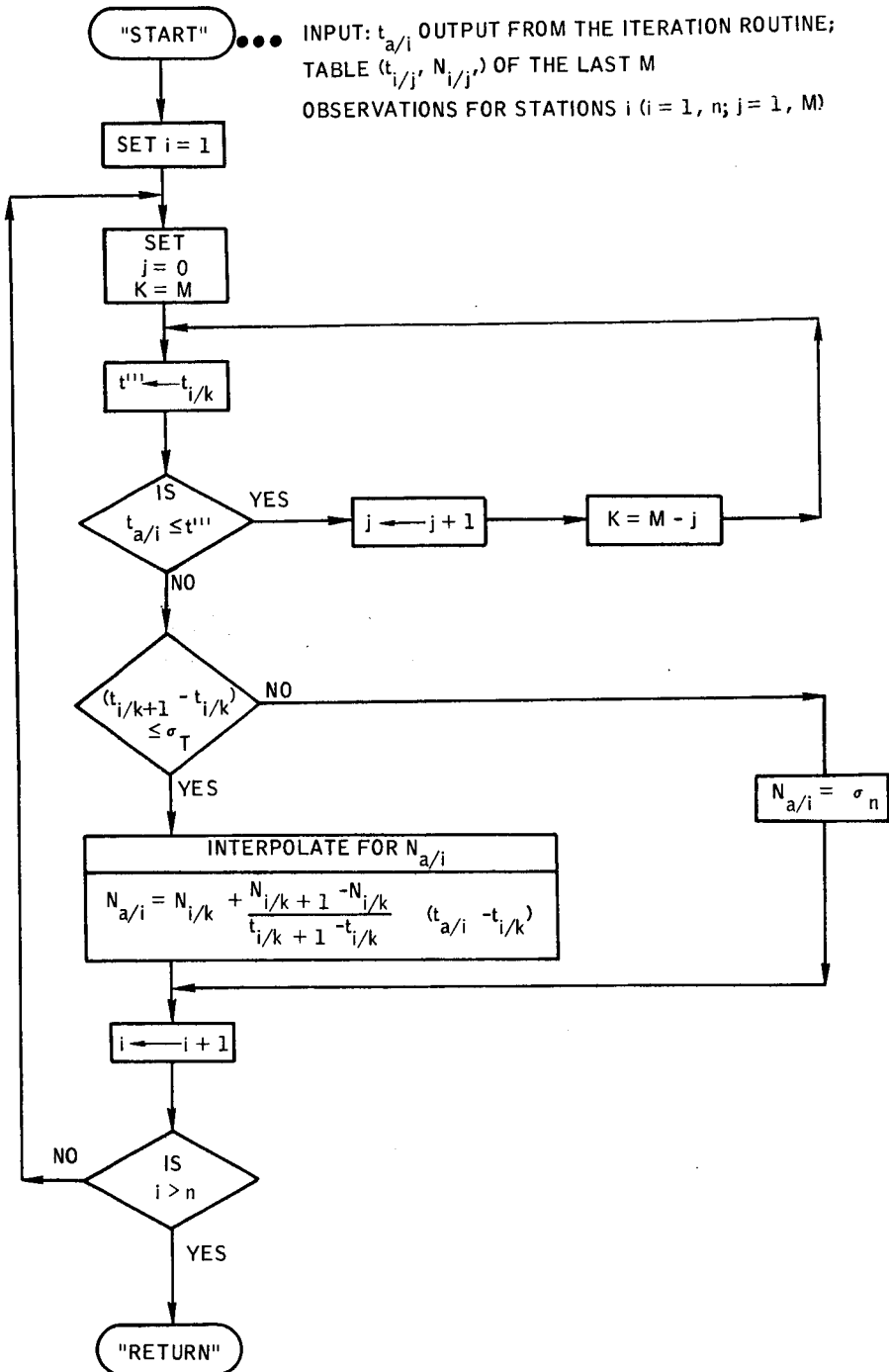
Flowchart 2.- Iteration subroutine (I).



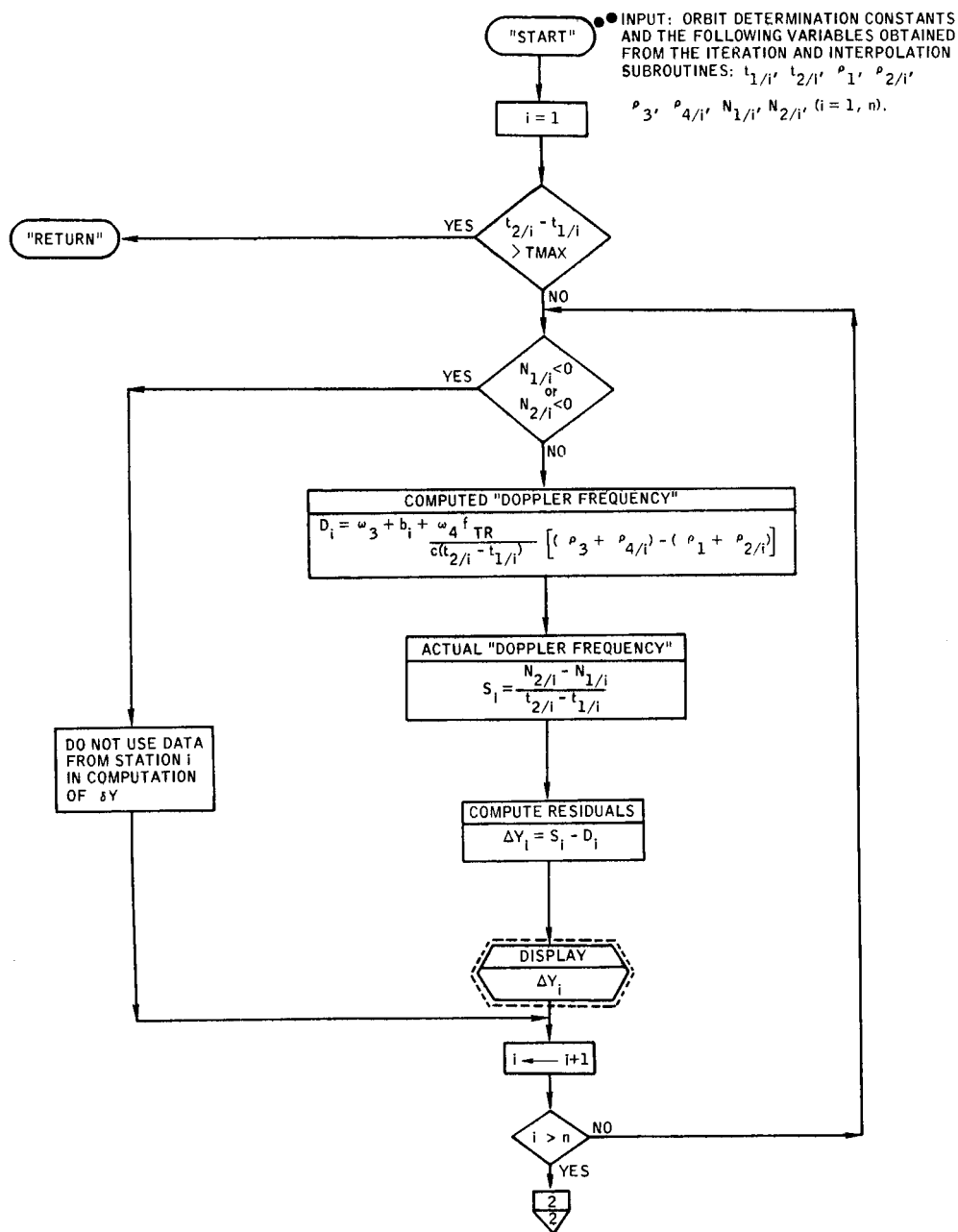
Flowchart 2.- Continued.



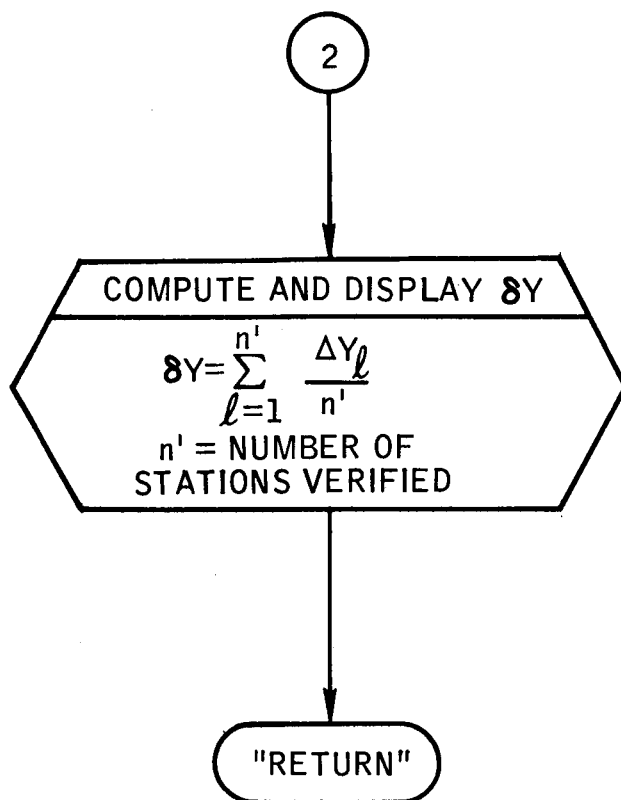
Flowchart 2,.- Concluded.



Flowchart 3.- Interpolation subroutine (II).



Flowchart 4.- Residual subroutine (III).



Flowchart 4.- Concluded.

APPENDIX  
CONSTANTS AND VARIABLES USED IN SELECTING  
AND VERIFYING USBS DOPPLER DATA SOURCES



## APPENDIX

CONSTANTS AND VARIABLES USED IN  
SELECTING AND VERIFYING USBS DOPPLER DATA SOURCES

ORBIT DETERMINATION CONSTANTS

|          |  |
|----------|--|
| $t_{LO}$ | time from lift-off to midnight of the day of LM ascent, sec (UTC)  |
| $T$      | number of Julian centuries (J.C.) which have elapsed from Jan. 0.5, 1900 to midnight preceding the computed time, $t'$ , of the Doppler observation at a station, J.C. |
| $\omega$ | rotational rate of earth, $7.29211506 \times 10^{-5}$ rad/sec  |
| $c$      | velocity of light, $9.835712 \times 10^8$ fps  |
| $b_i$    | bias for station on the actual Doppler observables (previously solved for in a preascent routine), Hz  |

STATION CHARACTERISTIC CONSTANTS

The following constants are found on the station characteristics tape.

|                              |  |
|------------------------------|--|
| $r \cos \phi' + h \cos \phi$ | projection of position vector of the station into the x-y plane in the ECI coordinate system |
| $r \sin \phi' + h \sin \phi$ | z coordinate of station in ECI system  |

where  $r$  radius to ellipsoid below station, ft

$\phi'$  geocentric latitude of station, rad

$\phi$  geodetic latitude of station, rad

$h$  height of station above ellipsoid, ft

|           |  |
|-----------|--|
| $\bar{n}$ | radio refractivity $\times 10^{-6}$ , nd   |
| $K$       | decay constant calculated from radio refractivity with an exponential reference atmosphere, nd |
| $\lambda$ | longitude of station, rad  |
| $f_{TR}$  | transmitted frequency, $2.101802 \times 10^9$ Hz   |

#### MISCELLANEOUS CONSTANTS

The following constants were taken from reference 1 (pp. 69-91).

|                            |  |
|----------------------------|--|
| TR                         | translation from selenocentric to ECI coordinates computed using the JPL lunar ephemeris tape, ft                |
| $g_1$                      | 1.7399358937 rad   |
| $g_2$                      | 628.33195102 rad/J.C.  |
| $g_3$                      | .00000676 rad/(J.C.) <sup>2</sup>  |
| $\delta\psi \cos \epsilon$ | equation of the equinoxes, updated when the (NP) matrix is updated (see ref. 1, p. 74), rad                      |
| DT                         | difference between UT1 and UTC, DT = UT1 - UTC, sec  |
| $\omega_3$                 | a bias introduced by the tracking equipment that guarantees that the shift will never become negative, $10^6$ Hz |
| $\omega_4$                 | a multiplying factor which adjusts the signal at the spacecraft, $\frac{240}{221}$ nd                            |

#### PROGRAM SWITCHES AND INDICES

|   |                                  |
|---|----------------------------------|
| i | station number, where $i = 1, n$ |
|---|----------------------------------|

|      |   |
|------|---|
| n    | number of stations  |
| n'   | number of stations verified; n' must be $\geq 2$  |
| M    | number of observations copied into the tables to be interpolated. $M = 100$ if the Doppler sample rate is 10/sec and $M = 50$ if the Doppler sample rate is 5/sec. That is, the observations encompass a time arc of 10 seconds |
| LSW  | switch set by the identification code of the telemetry vector: $LSW = 0$ for PGNCS vector, $LSW \neq 0$ for AGS.  |
| KSWP | switch which denotes the PGNCS vector: $KSWP = 0$ for first PGNCS vector, $KSWP \neq 0$ thereafter.   |
| KSWA | switch which denotes the AGS vector: $KSWA = 0$ for first AGS vector, $KSWA \neq 0$ thereafter.   |
| ISW  | switch which denotes uplink or downlink range computation: $ISW = 0$ for uplink range computation, $ISW \neq 0$ for downlink computations.  |
| LOOP | counter used in the iteration subroutine:   |
| a    | index used for current values and computations  |
| b    | index used for previous values and computations   |

#### PROGRAM VARIABLES

The following program variables are listed in the order in which they appear.

##### Main Program

|           |  |
|-----------|--|
| $t_a$     | time tag (g.e.t.) of a PGNCS or AGS telemetry vector, sec (UTC)  |
| $R(t_a)$  | selenocentric position vector of vehicle (PGNCS or AGS telemetry vector) in mean NBY selenocentric coordinates, ft |
| $t_{i/j}$ | time array ( $j = 1, M$ ) consisting of the last $M$ observation times at station $i$ , sec                        |

$N_{i/j}$  an array of raw, non-destruct Doppler counts recorded at station  $i$  at each  $t_{i/j}$ , counts

#### Iteration Subroutine

$R'(t_a)$  true-of-date ECI position vector of the vehicle at  $t_a$ , ft

$t_a^*$  time the signal would be transmitted from station 1 to arrive at the vehicle at  $t_a$ , sec

$\rho, \rho', t', t''$  temporary storage for use in iteration routine

$R_s(t')$  ECI true-of-date position vector of a station (either transmitter or receiver) at the computed transmission time  $t'$ , ft

$R_s(t_a)$  initial guess of  $R_s(t')$ , ft

$\bar{\rho}$  vector difference between  $R'(t_a)$  and  $R_s(t')$ , ft

$\rho_a^*$  uplink range from the transmitting station at  $t'$  to the vehicle at  $t_a$ , ft

$t_{a/i}$  time at which the signal would arrive at station  $i$  if it had left the vehicle at  $t_a$ , sec

$\rho_{a/i}$  downlink range from vehicle at  $t_a$  to station  $i$  at  $t_{a/i}$  ( $i = 1, n$ ), ft

$\theta$  angle between true-of-date position of Aries and Greenwich, rad

$\lambda'$  right ascension of station:  $\lambda' = \lambda + \theta$ , rad

#### Interpolation Subroutine

$N_{a/i}$  an interpolated N-count situated between  $N_{i/k}$  and  $N_{i/k+1}$ , counts

$\sigma_T$  the maximum time interval over which the Doppler data may be interpolated, sec

$\sigma_N$ dummy negative value for  $N_{a/i}$ , counts

## Residual Computation Subroutine

$$\left. \begin{array}{l} P t_{b/i} \\ P \rho_{b/i} \\ P \rho_b^* \\ P N_{b/i} \end{array} \right\}$$

temporary storage of previous values of  $t_{a/i}$ ,  $\rho_{a/i}$ ,  $\rho_a^*$ , and  $N_{a/i}$ , respectively, resulting from a PGNCS telemetry vector

$$\left. \begin{array}{l} A t_{b/i} \\ A \rho_{b/i} \\ A \rho_b^* \\ A N_{b/i} \end{array} \right\}$$

temporary storage of previous values of  $t_{a/i}$ ,  $\rho_{a/i}$ ,  $\rho_a^*$ , and  $N_{a/i}$ , respectively, resulting from an AGS telemetry vector.

TMAX

the maximum time interval between two successive vectors from the same source for which a Doppler residual may be computed, sec

 $\Delta Y_i$ 

residual for station  $i$  for a vector source (actual minus computed), Hz

 $\delta Y$ 

average value of the  $\Delta Y_i$ , ( $i = 1, n$ ), Hz

$$\left. \begin{array}{l} t_1 \\ t_2 \end{array} \right\}$$

successive computed observation times for a station, sec

$$\left. \begin{array}{l} \rho_1 \\ \rho_3 \end{array} \right\}$$

uplink ranges computed for  $t_1$  and  $t_2$  respectively, ft

$$\left. \begin{array}{l} \rho_2 \\ \rho_4 \end{array} \right\}$$

downlink ranges computed for  $t_1$  and  $t_2$  respectively, ft

$\left. \begin{array}{l} N_1 \\ N_2 \end{array} \right\}$ 

interpolated values of N-count at  $t_1$  and  $t_2$ ,  
counts

 $D_i$ 

computed Doppler frequency at station  $i$ , Hz

 $S_i$ 

observed Doppler frequency shift over

$t_{a/i} - t_{b/i}$ , Hz

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